General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MPC4

Unit Pure Core 4

Thursday 14 June 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 (a) (i) Express $\dfrac{5x-6}{x(x-3)}$ in the form $\dfrac{A}{x}+\dfrac{B}{x-3}$. *(2 marks)*

(ii) Find $\displaystyle\int \frac{5x-6}{x(x-3)}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Given that

$$4x^3+5x-2=(2x+1)(2x^2+px+q)+r$$

find the values of the constants p, q and r. *(4 marks)*

(ii) Find $\displaystyle\int \frac{4x^3+5x-2}{2x+1}\,\mathrm{d}x$. *(3 marks)*

---

2 (a) Express $\sin x - 3\cos x$ in the form $R\sin(x-\alpha)$, where $R>0$ and $0° < \alpha < 90°$, giving your value of α to the nearest $0.1°$. *(3 marks)*

(b) Hence find the values of x in the interval $0° < x < 360°$ for which

$$\sin x - 3\cos x + 2 = 0$$

giving your values of x to the nearest degree. *(4 marks)*

---

3 (a) Find the binomial expansion of $(1+4x)^{\frac{1}{2}}$ up to and including the term in x^2. *(2 marks)*

(b) (i) Find the binomial expansion of $(4-x)^{-\frac{1}{2}}$ up to and including the term in x^2. *(3 marks)*

(ii) State the range of values of x for which the expansion in part **(b)(i)** is valid. *(1 mark)*

(c) Find the binomial expansion of $\sqrt{\dfrac{1+4x}{4-x}}$ up to and including the term in x^2. *(2 marks)*

4 The value, £V, of an initial investment, £P, at the end of n years is given by the formula

$$V = P\left(1 + \frac{r}{100}\right)^n$$

where r% per year is the fixed interest rate.

Mr Brown invests £1000 in Barcelona Bank at a fixed interest rate of 3% per year.

(a) (i) Find the value of Mr Brown's investment at the end of 5 years. Give your value to the nearest £10. *(1 mark)*

(ii) The value of Mr Brown's investment will first exceed £2000 after N complete years.

Find the value of N. *(3 marks)*

(b) Mrs White invests £1500 in Bilbao Bank at a fixed interest rate of 1.5% per year. Mr Brown and Mrs White invest their money at the same time. The value of Mr Brown's investment will first exceed the value of Mrs White's investment after T complete years.

Find the value of T. *(4 marks)*

---

5 A curve is defined by the parametric equations

$$x = 2\cos\theta, \qquad y = 3\sin 2\theta$$

(a) (i) Show that

$$\frac{dy}{dx} = a\sin\theta + b\operatorname{cosec}\theta$$

where a and b are integers. *(4 marks)*

(ii) Find the gradient of the normal to the curve at the point where $\theta = \frac{\pi}{6}$. *(2 marks)*

(b) Show that the cartesian equation of the curve can be expressed as

$$y^2 = px^2(4 - x^2)$$

where p is a rational number. *(3 marks)*

---

6 A curve is defined by the equation $9x^2 - 6xy + 4y^2 = 3$.

Find the coordinates of the two stationary points of this curve. *(8 marks)*

Turn over ▶

7 The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 0 \\ -2 \\ q \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, where q is an integer.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$.

The lines l_1 and l_2 intersect at the point P.

(a) Show that $q = 4$ and find the coordinates of P. *(3 marks)*

(b) Show that l_1 and l_2 are perpendicular. *(1 mark)*

(c) The point A lies on the line l_1 where $\lambda = 1$.

(i) Find AP^2. *(2 marks)*

(ii) The point B lies on the line l_2 so that the right-angled triangle APB is isosceles.

Find the coordinates of the two possible positions of B. *(6 marks)*

---

8 (a) A water tank has a height of 2 metres. The depth of the water in the tank is h metres at time t minutes after water begins to enter the tank. The rate at which the depth of the water in the tank increases is proportional to the difference between the height of the tank and the depth of the water.

Write down a differential equation in the variables h and t and a positive constant k.

(You are not required to solve your differential equation.) *(3 marks)*

(b) (i) Another water tank is filling in such a way that t minutes after the water is turned on, the depth of the water, x metres, increases according to the differential equation

$$\frac{dx}{dt} = \frac{1}{15x\sqrt{2x-1}}$$

The depth of the water is 1 metre when the water is first turned on.

Solve this differential equation to find t as a function of x. *(8 marks)*

(ii) Calculate the time taken for the depth of the water in the tank to reach 2 metres, giving your answer to the nearest 0.1 of a minute. *(1 mark)*